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Light-front approach for pentaquark strong decays

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Abstract: Assuming the two diquark structure for the pentaquark state as advocated in the Jaffe-Wilczek model, we study the strong decays of light and heavy parity-even pentaquark states using the light-front quark model in conjunction with the spectator approximation. The narrowness of the Θ^+ width is ascribed to the p-wave configuration of the diquark pair. Taking the Θ^+ width as a benchmark, we estimate the rates of the strong decays $\Xi_{3/2}^{--} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Sigma_{5c}^0 \to D_s^- p$, $D_{s0}^{*-} p$ and $\Xi_{5c}^0 \to D_s^-\Sigma^+, D_{s0}^{*-}\Sigma^+$ with Σ_{5c}, Ξ_{5c} being antisextet charmed pentaquarks and D_{s0}^* a scalar strange charmed meson. The ratio of $\Gamma(\mathcal{P}_c \to \mathcal{B}D_{s0}^*)/\Gamma(\mathcal{P}_c \to \mathcal{B}D_s)$ is very useful for verifying the parity of the antisextet charmed pentaquark $\mathcal{P}_c$. It is expected to be of order unity for an even parity $\mathcal{P}_c$ and much less than one for an odd parity pentaquark.

Keywords: QCD, Phenomenological Models.

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Contents

1. Introduction

The recent discovery of an exotic Θ^+ baryon with $S = +1$ by LEPS at SPring-8 [1], subsequently confirmed by many other groups [2]–[11], led to a renewed interest in hadron spectroscopy and promoted a re-examination of the QCD implications for exotic hadrons (for a review of pentaquarks, see [12]). The mass of the Θ^+ is of order 1535 MeV and its width is less than 10 MeV from direct observations and can be as narrow as 1 MeV from the analysis of K-deuteron scattering data [13] and it is most likely an isosinglet. The $I = 3/2$ exotic pentaquark $\Xi^{--}_{3/2}$ with a mass of 1862 ± 2 MeV and a width smaller than 18 MeV was observed by NA49 [14] (see also [15] for a critical discussion). In spite of the confirmation of the Θ^+ from several experiments, all current experimental signals are weak and the significance is only of 4–6 standard deviations. Indeed, there exist many null results for the pentaquark search from [16]–[26]. An effort for understanding why the Θ^+ is seen in some experiments but not in others has been made in [27]. The pentaquark candidate signals must be established beyond any doubt by increasing the experimental statistics.

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The Θ^+ mass is expected to be of order 1900 MeV for an s-wave ground state with odd parity and 2200 MeV for a p-wave state with even parity in the conventional uncorrelated quark model. The width is at least of order several hundred MeV as the strong decay $\Theta^+ \to KN$ is Okubo-Zweig-Iizuka (OZI) super-allowed. Therefore, within the naive uncorrelated quark model one cannot understand why Θ^+ is anomalously light and why its width is so narrow. This hints at a possible correlation among various quarks; two or three quarks could form a cluster. Several quark cluster models have been proposed in the past [28]–[30]. For example, Jaffe and Wilczek [28] advocated a two diquark picture in which the Θ^+ is a bound state of an $\bar{s}$ quark with two (ud) diquarks. The diquark is a highly correlated spin-zero object and is in a flavor anti-triplet and color anti-triplet state. The parity of Θ^+ is flipped from the negative, as expected in the naive quark model, to the positive owing to the orbital angular momentum between two diquarks. The even parity of the Θ^+ is in agreement with the prediction of the chiral soliton model [31]. Note that two of previous lattice calculations imply a negative parity for the Θ^+ [32, 33]. However, based on the Jaffe-Wilczek picture to construct the interpolating operators, a recent quenched lattice QCD calculation with exact chiral symmetry yields a positive parity for the pentaquark states Θ^+, $\Xi_{3/2}^{--}$ [34] and for charmed pentaquarks to be discussed below [35].

It is natural to consider the heavy flavor analogs Θ_c^0 and Θ_b^+ of Θ^+ by replacing the $\bar{s}$ quark in Θ^+ by the heavy antiquark $\bar{c}$ and $\bar{b}$, respectively. Whether the mass of the heavy pentaquark state is above or below the strong-decay threshold has been quite controversial. Very recently, a narrow resonance in $D^{*-}p$ and $D^{*+}\bar{p}$ invariant mass distributions was reported by the H1 Collaboration [36]. It has a mass of $3099 \pm 3 \pm 5$ MeV and a gaussian width of 12 ± 3 MeV and can be identified with the spin 1/2 or 3/2 charmed pentaquark baryon. However, there are also several null results reported by ZEUS [37], ALEPH [20] and FOCUS [25]. Although the state observed by H1 is about 300 MeV higher than the DN threshold, it is possible that the observed H1 pentaquark is a chiral partner of the yet undiscovered ground state Θ_c^0 with opposite parity and a mass of order 2700 MeV as implied by several model estimates [38]. The latter pentaquark can be discovered only through its weak decay [39]. Note that the theoretical estimates of Θ_c mass are controversial even within the Jaffe-Wilczek picture: The original estimate made by Jaffe and Wilczek is below the $D^{(*)}p$ threshold [28], while other calculations in [35, 40] that take into account hyperfine interactions between the anti-charmed quark and the two diquarks yield a charmed pentaquark mass above the strong-decay threshold. The latter is also preferred by a recent QCD sum rule calculation [41]. Given the situation, it is therefore interesting to consider the strong decays of charmed pentaquarks as well.

In the Jaffe-Wilczek model, there exist parity-even antisextet and parity-odd triplet heavy pentaquarks containing a single heavy antiquark $\bar{c}$ or $\bar{b}$ and they are all truly exotic. The heavy pentaquark baryons in the $\mathbf{3}_f$ representation are lighter than the $\bar{\mathbf{6}}_f$ ones due to the lack of orbital excitation and therefore may be stable against strong decays [42, 35]. Consequently, it becomes important to study the weak decays of triplet heavy pentaquarks [39, 43]. In [39] we have employed the relativistic light-front (LF) approach to study the heavy pentaquark weak decays. It is found that the weak transition form factors thus obtained are consistent with heavy quark symmetry [44]–[47].

The light-front model allows us to study the transition form factors and their momentum dependence. Furthermore, large relativistic effects which may manifest near the maximum large recoil, i.e. $q^2 = 0$, are properly taken into account in the light-front framework. In this work we shall extend the formalism to pentaquark strong decays. The strong decays of pentaquarks have been studied in [48]–[55] and they can be classified into

$$\begin{aligned} &(a) \quad \mathcal{P}(\overline{\mathbf{10}}) \to \mathcal{B}(\mathbf{8}) + M\,, \\ &(b) \quad \mathcal{P}_Q(\bar{\mathbf{6}}) \to \mathcal{B}(\mathbf{8}) + M_Q\,, \end{aligned} \tag{1.1}$$

where $\mathcal{P}_{(Q)}$ denotes a generic (heavy) pentaquark baryon, $\mathcal{B}(\mathbf{8})$ stands for the usual octet baryon made of three quarks and $M_{(Q)}$ is a (heavy) meson. Examples are $\Theta^+ \to pK^0$, $\Xi^{--} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Sigma^0_{5c} \to D^-_s p$, $\Xi^0_{5c} \to \overline{D}^0\Xi^0, D^-_s\Sigma^+, D^{*-}_{s0}\Sigma^+$. Of course, whether the above-mentioned strong decays are kinematically allowed or not depends on the (heavy) pentaquark masses. It is interesting to understand why the Θ^+ width is much smaller than a typical strong decay width. We find that the narrowness of the Θ^+ width is most likely ascribed to the p-wave configuration of its constituent diquark pair. It is important to note that the scalar charmed meson D^*_{s0} is experimentally found to have a mass of order 2317 MeV [56, 57], which is considerably lighter than expected from potential models [58]. Therefore, the $D^*_{s0}\mathcal{B}$ threshold is not far from the $D_s\mathcal{B}$ one, rendering the study of the decay $\mathcal{P}_c \to D^*_{s0}\mathcal{B}$ interesting. Indeed, since the parities of D^*_{s0} and $\mathcal{P}_c$ (in the Jeffe-Wilczek model) are the same, the $D^*_{s0}\mathcal{B}$ final state can be in a s-wave configuration. Thus it is not subject to a suppression near the threshold and hence can have a sizable decay rate compared to $\mathcal{P}_c \to D_s\mathcal{B}$. This could be useful for measuring the parity of $\mathcal{P}_c$.

The layout of the present paper is organized as follows. In section 2 we present a study of the pentaquark transitions within the light-front quark model and derive the analytic expressions for form factors. Numerical results for form factors and examples of strong decays of light and heavy pentaquark baryons are worked out in section 3. Conclusion is given in section 4 followed by the appendix devoted to various baryon and meson Clebsch-Gordan coefficients.

2. Formalism of a light-front model for pentaquarks

In this section we shall focus on the hadronic strong decays of light and heavy pentaquarks within the light-front approach and the Jaffe-Wilczek model. In this study we need to use pentaquark and meson vertex functions. We shall consider the mesonic case first as it is simpler. Readers who are not interested in the technical details of vertex functions can skip directly to section 2.2.

2.1 Vertex functions in the light-front approach

2.1.1 Vertex functions for mesons

In the conventional light-front approach, a meson bound state consisting of a quark q_1 and an antiquark $\bar{q}_2$ with the total momentum P and spin J can be written as (see, for

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example, [59] for odd-parity and [60] for even-parity mesons)

$$|M(P,\, {}^{2S+1}L_J, J_z)\rangle = \int \{d^3p_1\}\{d^3p_2\}\ 2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2)\times \\ \times \sum_{\lambda_1,\lambda_2,\alpha,a,b} \Psi^{JJ_z}_{LS}(x_1,x_2,k_{1\perp},k_{2\perp})\ M^b_a|(q^c)^{a\alpha}(p_1,\lambda_1)(\bar{q}^c)_{b\alpha}(p_2,\lambda_2)\rangle\,, \tag{2.1}$$

where a, b are flavor indices,[1] α is the color index, M^b_a is a normalized matrix element characterizing the meson $\mathrm{SU}_f(3)$ quantum number (see the appendix for details), p_1, p_2 are the on-mass-shell light-front momenta,

$$\tilde{p} = (p^+, p_\perp)\,, \qquad p_\perp = (p^1, p^2)\,, \qquad p^- = \frac{m^2+p_\perp^2}{p^+}\,, \tag{2.2}$$

and

$$\begin{aligned} \{d^3p\} &\equiv \frac{dp^+d^2p_\perp}{2(2\pi)^3}\,, \\ |q^c(p_1,\lambda_1)\bar{q}^c(p_2,\lambda_2)\rangle &= b^\dagger_{\lambda_1}(p_1)d^\dagger_{\lambda_2}(p_2)|0\rangle\,, \\ \{b_{\lambda'}(p'), b^\dagger_\lambda(p)\} &= \{d_{\lambda'}(p'), d^\dagger_\lambda(p)\} = 2(2\pi)^3\ \delta^3(\tilde{p}'-\tilde{p})\ \delta_{\lambda'\lambda}\,. \end{aligned} \tag{2.3}$$

Note that we use the charge conjugated fields for quarks. For example, we shall use $|c^c(p_1,\lambda_1)\bar{d}^c(p_2,\lambda_2)\rangle$ in eq. (2.1) for the D^- meson. The reason for using the charged conjugated field will become clear later.

In terms of the light-front relative momentum variables $(x, p_\perp)$ defined by

$$\begin{aligned} &p_1^+ = x_1P^+\,, \qquad p_2^+ = x_2P^+\,, \qquad x_1+x_2 = 1\,, \\ &p_{1\perp} = x_1P_\perp + k_{1\perp}\,, \qquad p_{2\perp} = x_2P_\perp + k_{2\perp}\,, \qquad k_\perp = k_{1\perp} = -k_{2\perp}\,, \end{aligned} \tag{2.4}$$

the momentum-space wave-function $\Psi^{JJ_z}_{LS}$ for a ${}^{2S+1}L_J$ meson can be expressed as

$$\Psi^{JJ_z}_{LS}(x_1,x_2,k_{1\perp},k_{2\perp}) = \frac{1}{\sqrt{N_c}}\langle LS;L_zS_z|LS;JJ_z\rangle R^{SS_z}_{\lambda_1\lambda_2}(x,k_\perp)\ \psi_{LL_z}(x,k_\perp)\,, \tag{2.5}$$

where $x \equiv x_2$, $\varphi_{LL_z}(x,k_\perp)$ describes the momentum distribution of the constituent quarks in the bound state with the orbital angular momentum L, $\langle LS;L_zS_z|LS;JJ_z\rangle$ is the corresponding Clebsch-Gordan coefficient and $R^{SS_z}_{\lambda_1\lambda_2}$ constructs a state of definite spin (S, S_z) out of light-front helicity (λ_1, λ_2) eigenstates. Explicitly [61, 62],

$$R^{SS_z}_{\lambda_1\lambda_2}(x,k_\perp) = \sum_{s_1,s_2}\langle\lambda_1|\mathcal{R}^\dagger_M(1-x,k_\perp,m_1)|s_1\rangle\langle\lambda_2|\mathcal{R}^\dagger_M(x,-k_\perp,m_2)|s_2\rangle\left\langle\frac{1}{2}\,\frac{1}{2};s_1s_2|\frac{1}{2}\frac{1}{2};SS_z\right\rangle, \tag{2.6}$$

where $|s_i\rangle$ are the usual Pauli spinors, and $\mathcal{R}_M$ is the Melosh transformation operator [61, 62]:

$$\begin{aligned} \langle s|\mathcal{R}_M(x,k_\perp,m_i)|\lambda\rangle &= \frac{\bar{u}_D(k_i,s)u(k_i,\lambda)}{2m_i} = -\frac{\bar{v}(k_i,\lambda)v_D(k_i,s)}{2m_i} \\ &= \frac{(m_i+x_i\bar{M}_0)\delta_{s\lambda} + i\vec{\sigma}_{s\lambda}\cdot\vec{k}_\perp\times\vec{n}}{\sqrt{(m_i+x_iM_0)^2+k_\perp^2}}\,, \end{aligned} \tag{2.7}$$

[1] Note that we use the field convention instead of the particle convention to denote the quantum numbers of the state, i.e. the state quantum number is defined according to the field creating the state.

with $u_{(D)}$, a Dirac spinor in the light-front (instant) form which has the expression

$$u_D(k,s) = \frac{\not k + m}{\sqrt{k^0+m}} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}, \qquad u(k,\lambda) = \frac{\not k + m}{\sqrt{2k^+}} \gamma^+ \gamma^0 \begin{pmatrix} \chi_\lambda \\ 0 \end{pmatrix}, \tag{2.8}$$

in the Dirac representation, $\vec{n} = (0,0,1)$, a unit vector in the z-direction, and

$$M_0^2 = \frac{m_1^2 + k_\perp^2}{x_1} + \frac{m_2^2 + k_\perp^2}{x_2}. \tag{2.9}$$

Note that $u_D(p,s) = u(p,\lambda)\langle\lambda|\mathcal{R}_M^\dagger|s\rangle$ and, consequently, the state $|q(p,\lambda)\rangle\langle\lambda|\mathcal{R}_M^\dagger|s\rangle$ transforms like $|q(p,s)\rangle$ under rotation, i.e. its transformation does not depend on its momentum. A crucial feature of the light-front formulation of a bound state, such as the one shown in eq. (2.1), is the frame-independence of the light-front wave function [63, 61]. Namely, the hadron can be boosted to any (physical) $(P^+,\, P_\perp)$ without affecting the internal variables $(x,\, k_\perp)$ of the wave function, which is certainly not the case in the instant-form formulation.

In practice it is more convenient to use the covariant form for $R^{SS_z}_{\lambda_1\lambda_2}$ [64]:

$$R^{SS_z}_{\lambda_1\lambda_2}(x,k_\perp) = \frac{1}{\sqrt{2}\ \widetilde{M}_0(M_0+m_1+m_2)}\ \bar{u}(p_1,\lambda_1)(\bar{\not P} + M_0)\Gamma\, v(p_2,\lambda_2), \tag{2.10}$$

with

$$\begin{aligned}
\widetilde{M}_0 &\equiv \sqrt{M_0^2 - (m_1-m_2)^2}\,, \\
\bar{P} &\equiv p_1 + p_2\,, \\
\varepsilon^\mu(\bar{P},\pm1) &= \left[\frac{2}{P^+}\vec{\varepsilon}_\perp(\pm1)\cdot\vec{P}_\perp,\, 0,\, \vec{\varepsilon}_\perp(\pm1)\right], \qquad \vec{\varepsilon}_\perp(\pm1) = \mp\frac{(1,\pm i)}{\sqrt{2}}\,, \\
\varepsilon^\mu(\bar{P},0) &= \frac{1}{M_0}\left(\frac{-M_0^2+P_\perp^2}{P^+}, P^+, P_\perp\right).
\end{aligned} \tag{2.11}$$

For the pseudoscalar and vector mesons, we have

$$\begin{aligned}
\Gamma_P &= \gamma_5 \qquad (\text{pseudoscalar}, L=0, S=0)\,, \\
\Gamma_V &= -\not\varepsilon(\bar{P},S_z) \qquad (\text{vector}, L=0, S=1)\,,
\end{aligned} \tag{2.12}$$

where

$$M_0 = e_1 + e_2\,, \qquad e_i = \sqrt{m_i^2 + k_\perp^2 + k_z^2}\,, \qquad k_z = \frac{x_1 M_0}{2} - \frac{m_1^2 + k_\perp^2}{2x_1 M_0}. \tag{2.13}$$

Applying equations of motion on spinors to eq. (2.10) leads to

$$\begin{aligned}
\bar{u}(p_1)(\bar{\not P} + M_0)\gamma_5\, v(p_2) &= (M_0+m_1+m_2)\bar{u}(p_1)\gamma_5\, v(p_2)\,, \\
\bar{u}(p_1)(\bar{\not P} + M_0)\not\varepsilon\, v(p_2) &= \bar{u}(p_1)[(M_0+m_1+m_2)\not\varepsilon - \varepsilon\cdot(p_1-p_2)]\, v(p_2)\,,
\end{aligned} \tag{2.14}$$

and $R^{SS_z}_{\lambda_1\lambda_2}$ is reduced to a more familiar form [64]. It is, however, more convenient to use the form shown in eq. (2.10) when extending to the p-wave meson case. Two remarks are in order. First, $p_1 + p_2$ is not equal to the meson's four-momentum in the conventional LF approach as both the quark and antiquark are on-shell. Second, the longitudinal polarization 4-vector $\varepsilon^\mu(\bar{P},0)$ given above is not exactly the same as that of the vector meson and we have $\varepsilon(\bar{P},S_z)\cdot\bar{P}=0$. We normalize the meson state as

$$\langle M(P',J',J'_z)|M(P,J,J_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}'-\tilde{P})\delta_{J'J}\delta_{J'_zJ_z}\,, \tag{2.15}$$

so that

$$\int \frac{dx\, d^2k_\perp}{2(2\pi)^3}\, \psi^*_{L'L'_z}(x,k_\perp)\psi_{LL_z}(x,k_\perp) = \delta_{L'L}\ \delta_{L'_zL_z}\,. \tag{2.16}$$

Explicitly, we have

$$\begin{aligned}\psi_{LL_z}(x,k_\perp) &= \sqrt{\frac{dk_z}{dx}}\ \varphi_{LL_z}(x,k_\perp)\,, \qquad \frac{dk_z}{dx} = \frac{e_1e_2}{x_1x_2M_0}\,,\\ \varphi_{00}(x,k_\perp) &= \varphi(\vec{k},\beta)\,, \qquad \varphi_{1m}(x,k_\perp) = k_m\varphi_p(\vec{k},\beta)\,,\end{aligned} \tag{2.17}$$

where $k_m = -\vec{\varepsilon}(m)\cdot\vec{k} = \varepsilon(\bar{P},m)\cdot(p_1-p_2)/2$, or explicitly $k_{m=\pm1} = \pm(k_{\perp x}\pm ik_{\perp y})/\sqrt{2}$, $k_{m=0}=-k_z$ are proportional to the spherical harmonics Y_{1m} in the momentum space, and φ, φ_p are the distribution amplitudes of s-wave and p-wave mesons, respectively. There are several popular phenomenological light-front wave functions that have been employed to describe various hadronic structures in the literature. For a gaussian-like wave function, one has [59, 60]

$$\varphi(\vec{k},\beta) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\ \exp\left(-\frac{k_z^2+k_\perp^2}{2\beta^2}\right),\quad \varphi_p(\vec{k},\beta) = \sqrt{\frac{2}{\beta^2}}\ \varphi(\vec{k},\beta)\,. \tag{2.18}$$

The parameter β is expected to be of order $\Lambda_{\rm QCD}$ and will be specified later.

It is straightforward to obtain [64, 39]

$$\begin{aligned} iR^{00}_{\lambda_1\lambda_2}(x,k_\perp) &= \frac{i}{\sqrt{2}\ \widetilde{M}_0}\ \bar{u}(p_1,\lambda_1)\gamma_5\, v(p_2,\lambda_2)\,,\\ \langle 1S;L_zS_z|1S;00\rangle\, \varepsilon(\bar{P},L_z)\cdot\frac{p_1-p_2}{2}\, R^{SS_z}_{\lambda_1\lambda_2}(x,k_\perp) &= -\frac{\widetilde{M}_0}{2\sqrt{6}\ M_0}\bar{u}(p_1,\lambda_1)v(p_2,\lambda_2)\,,\end{aligned} \tag{2.19}$$

where $\langle 1S;L_zS_z|1S;00\rangle\, \varepsilon_\mu(\bar{P},L_z)\varepsilon_\nu(\bar{P},S_z) = -\varepsilon^*_\mu(\bar{P},S_z)\varepsilon_\nu(\bar{P},S_z)/\sqrt{3}$ have been made. Note that an overall phase i is assigned to the ${}^1S_0$ state to match the usual phase convention. Putting everything together, we have

$$\begin{aligned} |M(P,{}^1S_0,0)\rangle &= i\int\{d^3p_1\}\{d^3p_2\}\ 2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2)\,\times\\ &\quad\times\sum_{\lambda_1,\lambda_2,\alpha}\frac{M_a^b}{\sqrt{2N_c}\ \widetilde{M}_0}\ \bar{u}(p_1,\lambda_1)\gamma_5\, v(p_2,\lambda_2)\psi_{00}(x,k_\perp)\,\times\\ &\quad\times|(q^c)^{a\alpha}(p_1,\lambda_1)(\bar{q}^c)_{b\alpha}(p_2,\lambda_2)\rangle\,,\\ |M(P,{}^3P_0,0)\rangle &= -\int\{d^3p_1\}\{d^3p_2\}\ 2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2)\,\times\\ &\quad\times\sum_{\lambda_1,\lambda_2,\alpha}\frac{\widetilde{M}_0\ M_a^b}{2\sqrt{3\beta^2N_c}\ M_0}\bar{u}(p_1,\lambda_1)v(p_2,\lambda_2)\psi_{00}(x,k_\perp)\,\times\\ &\quad\times|(q^c)^{a\alpha}(p_1,\lambda_1)(\bar{q}^c)_{b\alpha}(p_2,\lambda_2)\rangle\,. \end{aligned} \tag{2.20}$$

Note that for heavy mesons with the $\bar{Q}q$ flavor content, where Q denotes a heavy quark, they transfer as $\mathrm{SU}_f(3)$ triplet states. Wave functions of these states are similar to those in the above equation, except that q^c is replaced by Q^c and M_a^b by M^b.

For the later purpose and for checking the phase convention, we shall consider the meson decay constants. For $J = 0$ mesons, the decay constants are defined by the matrix elements

$$\langle 0|\bar{q}_2^c\gamma_\mu\gamma_5 q_1^c|P(P)\rangle \equiv i f_P P_\mu\,, \qquad \langle 0|\bar{q}_2^c\gamma_\mu q_1^c|S(P)\rangle \equiv -f_S P_\mu\,, \tag{2.21}$$

where the P and S denote pseudoscalar and scalar $q_1^c\bar{q}_2^c$ mesons, respectively, and an additional minus sign before f_S is due to charge conjugation. Using the relation

$$\langle 0|(\bar{q}^c)^{b'\alpha'}\ \gamma^+(\gamma_5)\ q^c_{a'\alpha'}|(q^c)^{a\alpha}(p_1,\lambda_1)(\bar{q}^c)_{b\alpha}(p_2,\lambda_2)\rangle = \frac{N_c\delta^a_{a'}\delta^{b'}_b}{\sqrt{p_1^+p_2^+}}\bar{v}(p_2,\lambda_2)\gamma^+(\gamma_5)u(p_1,\lambda_1)\,, \tag{2.22}$$

and considering V^+ and A^+ matrix elements, we obtain [60]

$$\begin{aligned} f_P &= 2\frac{\sqrt{2N_c}}{16\pi^3}\int dx_2 d^2k_\perp \frac{1}{\sqrt{x_1x_2}\widetilde{M}_0}\left(m_1x_2+m_2x_1\right)\varphi(x_2,k_\perp)\,,\\ f_S &= 2\frac{\sqrt{2N_c}}{16\pi^3}\int dx_2 d^2k_\perp \frac{\widetilde{M}_0}{2\sqrt{3x_1x_2}M_0'}\left(m_1x_2-m_2x_1\right)\varphi_p(x_2,k_\perp)\,. \end{aligned} \tag{2.23}$$

It is easy to see that for $m_1 = m_2$, the scalar meson wave function is symmetric with respect to x_1 and x_2, and hence $f_S = 0$, as it should be [65, 39].

2.1.2 Vertex functions for pentaquarks

We adopt the Jaffe-Wilczek picture [28] for the pentaquark $\mathcal{P}_{(Q)}$ which has the quark flavor content $\bar{q}[q_1q_2][q_3q_4]$ ($\bar{Q}[q_1q_2][q_3q_4]$). Vertex functions for pentaquarks in the light-front approach is first formulated in [39]. For the purpose of the calculational convenience, we shall treat the antiquark $\bar{q}$ as a particle q^c instead of an antiparticle, i.e. we shall use the charge conjugated field [39]. The reason for this seemingly odd choice will become clear in later calculations.

The scalar diquark transforms as an anti-triplet in both color and flavor spaces. We use $\phi_{a\alpha}$, where a and α are flavor and color indices, respectively, to denote a diquark field. More explicitly, in the sense of color and flavor quantum numbers, we have

$$\phi_{a\alpha} \sim \epsilon_{\alpha\beta\gamma}\epsilon_{abc}[q^{b\beta}q^{c\gamma}]\,. \tag{2.24}$$

For example, we have $\phi_{3\alpha} \sim \epsilon_{\alpha\beta\gamma}[u^\beta d^\gamma]$. Note that ϕ is not an extrapolating field constructed from bilinear quark fields, instead it is considered as an effectively fundamental bosonic field to describe the degrees of freedom of the composite diquark system. In the Jaffe-Wilczek picture [28] the diquark pair in the even (odd) parity pentaquark is in a $L = 1$ (0) configuration.

In the light-front approach, the pentaquark bound state with the total momentum P, spin $J = 1/2$ and the orbital angular momentum of the diquark pair $L = 0, 1$ can be written as [39]

$$\begin{aligned}|\mathcal{P}(P, L, S_z)\rangle = \int & \{d^3p_1\}\{d^3p_2\}\{d^3p_3\}\, \frac{2(2\pi)^3}{\sqrt{P^+}} \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \times \\ & \times \sum_{\lambda_1, \alpha, \beta, \gamma, a, b, c} \Psi_L^{S_z}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}, \lambda_1)\; C_{\alpha\beta\gamma}(F_L)_{abc} \times \\ & \times \Big| (q^c)^{a\alpha}(p_1, \lambda_1) \phi^{b\beta}(p_2) \phi^{c\beta}(p_3) \Big\rangle\,, \end{aligned} \tag{2.25}$$

where α, β, γ and a, b, c are color and flavor indices, respectively, λ denotes helicity, p_1, p_2 and p_3 are the on-mass-shell light-front momenta,

$$\tilde{p} = (p^+, p_\perp)\,, \qquad p_\perp = (p^1, p^2)\,, \qquad p^- = \frac{m^2 + p_\perp^2}{p^+}, \tag{2.26}$$

and

$$\begin{aligned} &\{d^3p\} \equiv \frac{dp^+ d^2p_\perp}{2(2\pi)^3}, \quad \delta^3(\tilde{p}) = \delta(p^+)\delta^2(p_\perp)\,, \\ &\Big|(q^c)(p_1, \lambda_1)\phi(p_2)\phi(p_3)\Big\rangle = d^\dagger_{\lambda_1}(p_1) \frac{a^\dagger(p_2) a^\dagger(p_3)}{\sqrt{2}} |0\rangle\,, \\ &[a(p'), a^\dagger(p)] = 2(2\pi)^3\; \delta^3(\tilde{p}' - \tilde{p}),\ \{d_{\lambda'}(p'), d^\dagger_\lambda(p)\} = 2(2\pi)^3\; \delta^3(\tilde{p}' - \tilde{p})\; \delta_{\lambda'\lambda}\,. \end{aligned} \tag{2.27}$$

The coefficient $C_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}/\sqrt{6}$ is a normalized color factor and $(F_L)_{abc}$ is a normalized flavor coefficient obeying the relation

$$\begin{aligned} & C^{*\alpha'\beta'\gamma'}(F_L^*)^{a'b'c'} C_{\alpha\beta\gamma}(F_L)_{abc} \times \\ & \times \Big\langle (q'^c)_{a'\alpha'}(p_1', \lambda_1') \phi_{b'\beta'}(p_2') \phi_{c'\gamma'}(p_3') \Big| (q^c)^{a\alpha}(p_1, \lambda_1) \phi^{b\beta}(p_2) \phi^{c\gamma}(p_3) \Big\rangle = \\ & = 2^3 (2\pi)^9\; \delta^3(\tilde{p}_1' - \tilde{p}_1) \frac{1}{2} [\delta^3(\tilde{p}_2' - \tilde{p}_2)\delta^3(\tilde{p}_3' - \tilde{p}_3) + (-)^L \delta^3(\tilde{p}_2' - \tilde{p}_3)\delta^3(\tilde{p}_3' - \tilde{p}_2)] \delta_{\lambda_1'\lambda_1}\,. \end{aligned} \tag{2.28}$$

Note that $(F_L)_{abc}$ is (anti-)symmetric under $b \leftrightarrow c$ for $L = 1$ (0). For example, $(F_1)_{333} = 1$ is the only non-vanishing element of $(F_1)_{abc}$ in the Θ^+ case and further examples are given in the appendix. The second term on the right-hand-side of the above equation is similar to the first term but with the $p_2 \leftrightarrow p_3$ interchange and an additional $(-)^L$ factor. The factor of $(-)^L$ will be compensated by the corresponding wave function under the $p_2 \leftrightarrow p_3$ interchange (see the discussion after eq. (2.38)).

In terms of the light-front relative momentum variables $(x_i, k_{i\perp})$ for $i = 1, 2, 3$ defined by

$$\begin{aligned} p_i^+ &= x_i P^+\,, \qquad \sum_{i=1}^3 x_i = 1\,, \\ p_{i\perp} &= x_i P_\perp + k_{i\perp}\,, \qquad \sum_{i=1}^3 k_{i\perp} = 0\,, \end{aligned} \tag{2.29}$$

the momentum-space wave-function $\Psi_L^{S_z}$ can be expressed as

$$\Psi_L^{S_z}(x_i, k_{i\perp}, \lambda_1) = \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \left\langle L\frac{1}{2}; m s_1 \middle| L\frac{1}{2}; \frac{1}{2} S_z \right\rangle \times \\ \times \Phi_{Lm}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}) \,, \tag{2.30}$$

where $\Phi_{Lm}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp})$ describes the momentum distribution of the constituents in the bound state with the subsystem consisting of the particles 2 and 3 in the orbital angular momentum L, $L_z = m$ state, $\langle L\frac{1}{2}; m s_1 | 1\frac{1}{2}; \frac{1}{2} S_z \rangle$ is the corresponding Clebsch-Gordan coefficient and $\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle$ is the well normalized Melosh transform matrix element. Its explicit form is given in eq. (2.7). Note that internal variables in this case are defined as

$$\begin{aligned} M_0^2 &= \sum_{i=1}^{3} \frac{m_i^2 + k_{i\perp}^2}{x_i} \,, \qquad k_i = \left(\frac{m_i^2 + k_{i\perp}^2}{x_i M_0}, x_i M_0, \, k_{i\perp} \right) = (e_i - k_{iz}, e_i + k_{iz}, k_{i\perp}) \,, \\ M_0 &= e_1 + e_2 + e_3 \,, \qquad e_i = \sqrt{m_i^2 + k_{i\perp}^2 + k_{iz}^2} = \frac{x_i M_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0} \,, \\ k_{iz} &= \frac{x_i M_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0} \,. \end{aligned} \tag{2.31}$$

Although the same notation is applied to both meson and pentaquark internal quantities, one should be aware of their differences (cf. eqs. (2.9), (2.13) and (2.31)).

In practice it is more convenient to use the covariant form for the Melosh transform matrix element [39]

$$\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \left\langle L\frac{1}{2}; m s_1 \middle| L\frac{1}{2}; \frac{1}{2} S_z \right\rangle = \\ = \frac{1}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \, \bar{u}(p_1, \lambda_1) \Gamma_{Lm} u(\bar{P}, S_z) \,, \tag{2.32}$$

with

$$\begin{aligned} &\Gamma_{00} = 1 \,, \qquad \Gamma_{1m} = -\frac{1}{\sqrt{3}} \gamma_5 \, \not{\varepsilon}^*(\bar{P}, m) \,, \\ &\bar{P} \equiv p_1 + p_2 + p_3 \,, \\ &\varepsilon^\mu(\bar{P}, \pm 1) = \left[\frac{2}{P^+} \vec{\varepsilon}_\perp(\pm 1) \cdot \vec{P}_\perp, \, 0, \, \vec{\varepsilon}_\perp(\pm 1) \right] , \qquad \vec{\varepsilon}_\perp(\pm 1) = \mp \frac{(1, \pm i)}{\sqrt{2}}, \\ &\varepsilon^\mu(\bar{P}, 0) = \frac{1}{M_0} \left(\frac{-M_0^2 + P_\perp^2}{P^+}, P^+, P_\perp \right) , \end{aligned} \tag{2.33}$$

for pentaquark states with $L = 0$ or $L = 1$ diquark pairs. It should be remarked that in the conventional LF approach $\bar{P} = p_1 + p_2 + p_3$ is not equal to the baryon's four-momentum as all constituents are on-shell and consequently $u(\bar{P}, S_z)$ is not equal to $u(P, S_z)$; they satisfy different equations of motions $(\not{\bar{P}} - M_0) u(\bar{P}, S_z) = 0$ and $(\not{P} - M) u(P, S_z) = 0$. This is similar to the case of a vector meson bound state where the polarization vectors $\varepsilon(\bar{P}, S_z)$ and $\varepsilon(P, S_z)$ are different and satisfy different equations $\varepsilon(\bar{P}, S_z) \cdot \bar{P} = 0$ and $\varepsilon(P, S_z) \cdot P = 0$ [64]. Although $u(\bar{P}, S_z)$ is different than $u(P, S_z)$, they satisfy the relation

$$\gamma^+ u(\bar{P}, S_z) = \gamma^+ u(P, S_z) \,, \tag{2.34}$$

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followed from $\gamma^+\gamma^+ = 0$, $\bar{P}^+ = P^+$, $\bar{P}_\perp = P_\perp$. This is again in analogy with the case of $\varepsilon(\bar{P}, \pm 1) = \varepsilon(P, \pm 1)$. The above relation is useful in extracting transition form factors to be discussed later.

The pentaquark baryon state is normalized as

$$\langle \mathcal{P}(P', S'_z) | \mathcal{P}(P, S_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P}) \delta_{L'L} \delta_{S'_z S_z} \,, \tag{2.35}$$

so that (cf. eqs. (2.25), (2.28) and (2.30))

$$\begin{aligned} \int \left(\Pi_{i=1}^3 \frac{dx_i \, d^2 k_{i\perp}}{2(2\pi)^3} \right) 2(2\pi)^3 \delta \left(1 - \sum x_i \right) \delta^2 \left(\sum k_{i\perp} \right) \times \\ \times \Phi^*_{L'm'}(\{x\}, \{k_\perp\}) \Phi_{Lm}(\{x\}, \{k_\perp\}) = \delta_{L'L} \delta_{m'm} \,. \end{aligned} \tag{2.36}$$

Under the constraint of $1 - \sum_{i=1}^3 x_i = \sum_{i=1}^3 (k_i)_{x,y,z} = 0$, we have the expressions

$$\begin{aligned} \Phi_{Lm}(\{x\}, \{k_\perp\}) &= \sqrt{\frac{\partial(k_{2z}, k_{3z})}{\partial(x_2, x_3)}} \, \varphi_{00}(\vec{k}_1, \beta_1) \; \varphi_{Lm} \left(\frac{\vec{k}_2 - \vec{k}_3}{2}, \beta_{23} \right), \\ \frac{\partial(k_{2z}, k_{3z})}{\partial(x_2, x_3)} &= \frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}, \quad \varphi_{00}(\vec{k}, \beta) = \varphi(\vec{k}, \beta) \,, \quad \varphi_{1m}(\vec{k}, \beta) = k_m \varphi_p(\vec{k}, \beta) \,, \end{aligned} \tag{2.37}$$

where $k_m = -\vec{\varepsilon}(m) \cdot \vec{k} = \varepsilon(\bar{P}, m) \cdot p$, or explicitly $k_{m=\pm 1} = \pm (k_{\perp x} \pm i k_{\perp y})/\sqrt{2}$, $k_{m=0} = -k_z$, are proportional to the spherical harmonics Y_{1m} in momentum space, and φ, φ_p are the distribution amplitudes of s-wave and p-wave states, respectively. For a gaussian-like wave function, one has eq. (2.18) and [39]

$$\Phi_{1m}(\{x\}, \{k_\perp\}) = \sqrt{\frac{2}{\beta_{23}^2}} \frac{(k_2 - k_3)_m}{2} \Phi_{00}(\{x\}, \{k_\perp\}) \,. \tag{2.38}$$

It is easily seen that Φ_{Lm} picks up a factor of $(-)^L$ under the interchange of $k_2 \leftrightarrow k_3$. This will compensate the $(-)^L$ factor appearing in eq. (2.28). By virtue of eq. (2.32) it is straightforward to obtain

$$\begin{aligned} \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \left\langle 1 \frac{1}{2}; m s_1 \middle| 1 \frac{1}{2}; \frac{1}{2} S_z \right\rangle \frac{(k_2 - k_3)_m}{2} &= \\ = \frac{1}{2\sqrt{6(p_1 \cdot \bar{P} + m_1 M_0)}} \, \bar{u}(p_1, \lambda_1) \gamma_5 \left[\not{p}_2 - \not{p}_3 - \frac{\bar{P} \cdot (p_2 - p_3)}{M_0} \right] u(\bar{P}, S_z) \,, \end{aligned} \tag{2.39}$$

where the factor of $(k_2 - k_3)_m = \varepsilon(\bar{P}, m) \cdot (p_2 - p_3)$ comes from the wave function eq. (2.38) for the $L = 1$ case. The state $|\mathcal{P}(P, L, S_z)\rangle$ for a pentaquark $\mathcal{P}$ in the light-front model can now be obtained by using eqs. (2.25)–(2.39).

Putting everything together we have

$$\begin{aligned} |\mathcal{P}(P, L = 0, S_z)\rangle = \int \{d^3 p_1\} \{d^3 p_2\} \{d^3 p_3\} \, \frac{2(2\pi)^3}{\sqrt{P^+}} \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \times \\ \times \sum_{\lambda_1, \alpha, \beta, \gamma, a, b, c} \frac{\Phi_{00}(\{x\}, \{k_\perp\})}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \, \bar{u}(p_1, \lambda_1) u(\bar{P}, S_z) \times \\ \times C_{\alpha\beta\gamma} (F_{L=0})_{abc} \left| (q^c)^{a\alpha}(p_1, \lambda_1) \phi^{b\beta}(p_2) \phi^{c\beta}(p_3) \right\rangle , \end{aligned} \tag{2.40}$$

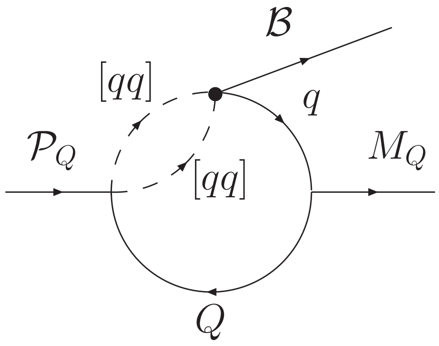

Figure 1: Feynman diagram for a typical $\mathcal{P}_Q \to M_Q\mathcal{B}$ transition with $\mathcal{B}$ being an octet baryon, where the spin-zero diquarks ($[qq] = [ud], [us], [ds]$) are denoted by dashed lines and the corresponding operator $\mathcal{O}_{\rm eff}$ (modelling the diquark pair to the $\mathcal{B}q$ transition as discussed in section 2.2 by $\bullet$.

for pentaquark states with $L = 0$ diquark pairs, and

$$\begin{aligned} |\mathcal{P}(P, L=1, S_z)\rangle = \int & \{d^3p_1\}\{d^3p_2\}\{d^3p_3\}\, \frac{2(2\pi)^3}{\sqrt{P^+}}\delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \times \\ & \times \sum_{\lambda_1,\alpha,\beta,\gamma,a,b,c} \frac{\Phi_{00}(\{x\},\{k_\perp\})}{\sqrt{12\ \beta_{23}^2(p_1\cdot \bar{P} + m_1 M_0)}} \times \\ & \times \bar{u}(p_1,\lambda_1)\gamma_5 \left[\not{p}_2 - \not{p}_3 - \frac{\bar{P}\cdot(p_2 - p_3)}{M_0}\right] u(\bar{P}, S_z) \times \\ & \times C_{\alpha\beta\gamma}(F_{L=1})_{abc} \left| (q^c)^{a\alpha}(p_1,\lambda_1)\phi^{b\beta}(p_2)\phi^{c\beta}(p_3) \right\rangle, \end{aligned} \tag{2.41}$$

for pentaquark states with $L = 1$ diquark pairs. Note that these vertex functions have been used to obtain weak transition form factors, which are consistent with the heavy quark symmetry [39].

2.2 Pentaquarks strong decays

2.2.1 Spectator approximation and the modelling of the $\phi\phi \to \mathcal{B}q$ sub-process

In a typical pentaquark decay to a meson and a baryon, the anti-quark is common to both pentaquark and the final state meson. To the leading order of the spectator approximation, the anti-quark can be considered as a spectator in the decay process depicted in figure 1. In this picture, there is a $\phi\phi \to \mathcal{B}q$ subprocess with $\phi\phi$ being a diquark pair and $\mathcal{B}$ a baryon. We use the effective hamiltonian [2]

$$H_{\rm eff} = \frac{g_{1\rm eff}}{M}\epsilon^{\alpha\beta\gamma}\epsilon^{abc}\overline{\mathcal{B}}^d_c\gamma_5(q^c)_{a\alpha}\phi_{b\beta}\phi_{d\gamma} + \frac{g_{2\rm eff}}{M^2}\epsilon^{\alpha\beta\gamma}\epsilon^{abc}\overline{\mathcal{B}}^d_c i\gamma^\mu\gamma_5(q^c)_{a\alpha}\phi_{b\beta}\partial_\mu\phi_{d\gamma} \tag{2.42}$$

to model (or mimic) the $\phi\phi \to \mathcal{B}q$ subprocess, where $M = \mathcal{O}(m_\phi, m_\mathcal{B})$ is a characteristic scale of the system. In general, the coupling constants $g_{1,2\rm eff}$ could have momentum

[2] Note that the γ_5 term is needed owing to the parity conservation for strong interactions and the presence of the q^c field.

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dependence. Since we are considering a soft process, we may regard $g_{1,2\mathrm{eff}}$ as averaged and effective coupling constants. Because the constituent quarks in octet baryons are in the s-wave configuration, it is necessary to bring the two diquarks in the pentaquark close together for interactions to induce a strong decay. Therefore, it is plausible to use local operators to approximate the effective hamiltonian.

The strong decay amplitude of a pentaquark can be approximated by

$$\begin{aligned} M(\mathcal{P}\to\mathcal{B}M) &\approx \langle \mathcal{B}M|H_{\mathrm{eff}}|\mathcal{P}\rangle \\ &= \langle \mathcal{B}|\ \bar{\mathcal{B}}^d_c|0\rangle \frac{g_{2\mathrm{eff}}}{M^2}\epsilon^{\alpha\beta\gamma}\epsilon^{abc}\langle M|i\gamma^\mu\gamma_5(q^c)_{a\alpha}\phi_{b\beta}\partial_\mu\phi_{d\gamma}|\mathcal{P}\rangle\,. \end{aligned} \tag{2.43}$$

As we shall see, only the $g_{2\mathrm{eff}}$ term in eq. (2.42) is relevant for the strong decays of even-parity pentaquarks under the spectator approximation. The antiquark common to the pentaquark and to the final state meson behaves as a spectator. Eq. (2.43) can be considered as an ansatz. In this work we shall estimate the matrix element $\langle M|(q^c)\phi\partial\phi|\mathcal{P}\rangle$ using the light-front approach. Once the coupling constant $g_{2\mathrm{eff}}$ is extracted from the process such as $\Theta^+\to KN$, we can apply it to estimate other $\mathcal{P}\to M\mathcal{B}$ strong decays.

In eq. (2.43) we have[3] $\langle\mathcal{B}|\ \bar{\mathcal{B}}^d_c|0\rangle=\bar{u}(P_{\mathcal{B}},S'_z)T^d_c$, where T^d_c is a traceless 3×3 matrix element corresponding to the emitted baryon SU(3) quantum number and its explicit form is given in the appendix. Defining

$$\mathcal{O}_{\mathrm{eff}}=i\epsilon^{\alpha\beta\gamma}\epsilon^{abc}\gamma^\mu\gamma_5(q^c)_{a\alpha}\phi_{b\beta}\partial_\mu\phi_{d\gamma}T^d_c\,. \tag{2.44}$$

we can recast eq. (2.43) to

$$M(\mathcal{P}\to M\mathcal{B})=\frac{g_{2\mathrm{eff}}}{M^2}\bar{u}(P_{\mathcal{B}},S'_z)\langle M|\mathcal{O}_{\mathrm{eff}}|\mathcal{P}\rangle\,. \tag{2.45}$$

From Lorentz covariance and SU(3) symmetry, we have the general expressions

$$\begin{aligned} \langle P(\mathbf{8})|\mathcal{O}_{\mathrm{eff}}|\mathcal{P}(\overline{\mathbf{10}})\rangle &= \epsilon^{ijk}\mathcal{P}_{imn}T^m_jP^n_k\ f(q^2)\,i\gamma_5u(P_{\mathcal{P}},S_z)\,,\\ \langle P_Q(\mathbf{3})|\mathcal{O}_{\mathrm{eff}}|\mathcal{P}_Q(\bar{\mathbf{6}})\rangle &= \epsilon^{ijk}(\mathcal{P}_Q)_{im}T^m_j(P_Q)_k\ f_Q(q^2)\,i\gamma_5u(P_{\mathcal{P}},S_z)\,,\\ \langle S_Q(\mathbf{3})|\mathcal{O}_{\mathrm{eff}}|\mathcal{P}_Q(\bar{\mathbf{6}})\rangle &= -\epsilon^{ijk}(\mathcal{P}_Q)_{im}T^m_j(S_Q)_k\ g_Q(q^2)u(P_{\mathcal{P}},S_z)\,, \end{aligned} \tag{2.46}$$

where f,f_Q,g_Q are form factors with dimension 2, $\mathcal{P}(\overline{\mathbf{10}})$ is an anti-decuplet pentaquark, $\mathcal{P}_Q(\bar{\mathbf{6}})$ is a heavy anti-sextet pentaquark, $P(\mathbf{8})$ is an octet pseudoscalar meson, $P_Q(\mathbf{3})$ is a heavy triplet pseudoscalar meson and $S_Q(\mathbf{3})$ is a heavy triplet scalar meson. Note that $\mathcal{P}_Q$ (P_Q,S_Q) in right hand side of the above equation is a 3×3 (3×1) matrix characterizing the SU(3) quantum numbers of the corresponding states; that is, $\mathcal{P}_{ijk}=(F_{L=1})_{ijk},(\mathcal{P}_Q)_{ij}=(F_{L=1})_{ij},P^l_m=M^l_m,(P_Q)_l=M_l,(S_Q)_l=M_l$. Our approach is consistent with the generic SU(3) approach [51]. Armed with the meson and pentaquark (phenomenological) wave functions (cf. eqs. (2.20), (2.40) and (2.41)), we are ready to estimate these form factors. To the end, we will gain more information than that based solely on flavor symmetry. For example, it will be interesting to see how the transition matrix elements involving different final state mesons, such as s-wave and p-wave ones, behave.

[3] Note that $|\mathcal{B}\rangle$ is normalized in the same way as eq. (2.35) and is different from the $|q\rangle$ normalization.

It is interesting to note that in the soft meson limit, the pentaquark decay amplitude can be related to the axial-vector matrix element $\langle \mathcal{B}|A_\mu|\mathcal{P}\rangle$. According to the action of the axial current there are two possible diagrams: an annihilation diagram and a transition one. The annihilation diagram has been considered in [54] and it is close to the one considered here as depicted in figure 1, while the transition diagram is the analogue of the so-called Z-graph. In the present framework, the Z-diagram is obtained by replacing the $\phi\phi \to \mathcal{B}q$ sub-process in figure 1 by the $\phi\phi q^c \to \mathcal{B}$ one. As in [60, 66, 39], we consider the $q^+ = 0$, $q_\perp \neq 0$ case where the Z-diagram contribution is absent [64, 59].

Note that eq. (2.46) is a spinor equation. The equation applies to each Dirac component. We shall follow [66, 39] to project out various form factors from the transition matrix elements. To extract form factors f, f_Q, g_Q, we apply the relation [67]

$$\frac{\bar u(P',S'_z)\gamma^+ u(P,S_z)}{2\sqrt{P^+P'^+}} = \frac{\bar u(\bar P',S'_z)\gamma^+ u(\bar P,S_z)}{2\sqrt{P^+P'^+}}$$
$$= \frac{u(\bar P',S'_z)\gamma^+ u(P,S_z)}{2\sqrt{P^+P'^+}} = \frac{\bar u(P',S'_z)\gamma^+ u(\bar P,S_z)}{2\sqrt{P^+P'^+}} = \delta_{S'_z S_z}\ , \quad (2.47)$$

which can be obtained by applying eqs. (2.8) and (2.34), and multiplying $\bar u(P,S_z)\gamma^+\gamma_5 (= \bar u(\bar P,S_z)\gamma^+\gamma_5)$ to the first two equations of eq. (2.46) from the left and $\bar u(P,S_z)\gamma^+ (= \bar u(\bar P,S_z)\gamma^+)$ to the last equation of eq. (2.46) from the left. The choice of this projection (or Dirac component) is due to the equality, eq. (2.47).

2.2.2 Even-parity Pentaquark to pseudoscalar and to scalar meson transitions

It is easy to derive the relation

$$\langle (q^c)_{a'\alpha'}(p'_1,\lambda'_1)(\bar q^c)^{b'\alpha'}(p'_2,\lambda'_2)|\gamma_5 (q^c)_{a''\alpha''}\ \phi_{b\beta''} i\partial_\mu \phi_{d''\gamma''}|(q^c)^{a\alpha}(p_1,\lambda_1)\phi^{b\beta}(p_2)\phi^{c\gamma}(p_3)\rangle =$$
$$= \frac{2(2\pi)^3}{\sqrt{2p'^+_2 p^+_2 p^+_3}}\gamma_5 v(p'_2,\lambda'_2)\delta^3(\tilde p'_1 - \tilde p_1)\delta_{\lambda_1\lambda'_1}\delta^{b'}_{a''}\delta^{\alpha}_{\alpha''}\delta^{a}_{a'}[p_{3\mu}\delta^{b}_{b''}\delta^{\beta}_{\beta''}\delta^{c}_{d''}\delta^{\gamma}_{\gamma''} + p_{2\mu}\delta^{b}_{d''}\delta^{\beta}_{\gamma''}\delta^{c}_{b''}\delta^{\gamma}_{\beta''}], \quad (2.48)$$

where the $1/\sqrt{2}$ factor is ascribed to the identical particles of $|\phi\phi\rangle$ and is included in the initial state as defined in eq. (2.27). Since we do not have an $\mathrm{SU}_f(3)$ singlet meson in the final state, a disconnected term, which occurs from the contraction of the final state quark-antiquark pair, is dropped from the above equation. For the even-parity pentaquark decay matrix element, the terms $\epsilon^{a''b''c''}\epsilon^{\alpha''\beta''\gamma''}T^{d''}_{c''}$ from $\mathcal{O}_{\rm eff}$ and $C_{\alpha\beta\gamma}(F_{L=1})_{abc}$ from $|\mathcal{P}\rangle$ will be contracted with the above equation. Since $(F_{L=1})_{abc}$ and $\epsilon^{\alpha\beta\gamma}$ are symmetric and anti-symmetric, respectively, in interchanging any of the two indices, we are led to a factor of $C_{\alpha\beta\gamma}\epsilon^{\alpha\beta\gamma}(F_{L=1})_{a'b''d''}\epsilon^{b'b''c''}T^{d''}_{c''}(p_3 - p_2)_\mu$ after contraction. It can be easily seen that the matrix element will be vanished if $\phi\phi$ rather than $\phi\partial\phi$ is employed in $\mathcal{O}_{\rm eff}$. This is the reason why only the $g_{2\rm eff}$ term in $H_{\rm eff}$ contributes.

For $\mathcal{P}(\overline{\mathbf{10}}) \to P(\mathbf{8})$ transitions, we have

$$\langle P(P')|\mathcal{O}_{\rm eff}|\mathcal{P}(P,S_z)\rangle = -i\int\{d^3p_1\}\{d^3p_2\}\frac{\epsilon^{lij}(F_{L=1})_{\rm min}T^n_j M^m_l}{2\sqrt{2\ \beta^2_{23}(p_1\cdot\bar P + m_1 M_0)P^+ p'^+_2 p^+_2 p^+_3 N_c \widetilde{M}'_0}}\times$$

$$\times\,(\not p_2-\not p_3)\gamma_5(\not p'_2-m'_2)\gamma_5(\not p_1+m_1)\gamma_5\left[\not p_2-\not p_3-\frac{\bar P\cdot(p_2-p_3)}{M_0}\right]\times$$
$$\times\,u(\bar P,S_z)\;\psi_{00}(x',k'_\perp)\;\Phi_{00}(\{x\},\{k_\perp\})\,, \tag{2.49}$$

with $p'_1=p_1$, $(p_2+p_3-p'_2-q)^+=(p_2+p_3-p'_2-q)_\perp=0$, $q^+=0$ or, equivalently,

$$x'_1=x_1\ \ (\text{or}\ \ x'_2=x_2+x_3)\,,\qquad k'_{1\perp}-x_1q_\perp=k_{1\perp}(=-k_{2\perp}-k_{3\perp})\,, \tag{2.50}$$

where $C_{\alpha\beta\gamma}\epsilon^{\alpha\beta\gamma}=\sqrt{6}$ and a relabelling of dummy indices has been made in eq. (2.49).

Likewise, for the case of $\mathcal{P}_Q(\bar{\mathbf{6}})\to P_Q(\mathbf{3})$ transitions, we have

$$\begin{aligned}\langle P_Q(P')|\mathcal{O}_{\rm eff}|\mathcal{P}_Q(P,S_z)\rangle\;=\;&-i\int\{d^3p_1\}\{d^3p_2\}\times\\&\times\frac{\epsilon^{lij}(F_{L=1})_{im}T^m_jM_l}{2\sqrt{2\;\beta^2_{23}(p_1\cdot\bar P+m_1M_0)P^+p'^+_2p^+_2p^+_3N_c\widetilde{M}'_0}}\times\\&\times(\not p_2-\not p_3)\gamma_5(\not p'_2-m'_2)\gamma_5(\not p_1+m_1)\gamma_5\times\\&\times\left[\not p_2-\not p_3-\frac{\bar P\cdot(p_2-p_3)}{M_0}\right]\times\\&\times u(\bar P,S_z)\;\psi_{00}(x',k'_\perp)\;\Phi_{00}(\{x\},\{k_\perp\})\,,\end{aligned} \tag{2.51}$$

while for the case of $\mathcal{P}_Q(\bar 6)\to S_Q(\mathbf{3})$ transitions,

$$\begin{aligned}\langle S_Q(P')|\mathcal{O}_{\rm eff}|\mathcal{P}_Q(P,S_z)\rangle\;=\;&\int\{d^3p_1\}\{d^3p_2\}\times\\&\times\frac{\epsilon^{lij}(F_{L=1})_{im}T^m_jM_l\widetilde{M}'_0}{4\sqrt{3\;\beta^2_{23}\beta'^2(p_1\cdot\bar P+m_1M_0)P^+p'^+_2p^+_2p^+_3N_cM'_0}}\times\\&\times(\not p_2-\not p_3)\gamma_5(\not p'_2-m'_2)(\not p_1+m_1)\gamma_5\times\\&\times\left[\not p_2-\not p_3-\frac{\bar P\cdot(p_2-p_3)}{M_0}\right]\times\\&\times u(\bar P,S_z)\;\psi_{00}(x',k'_\perp)\;\Phi_{00}(\{x\},\{k_\perp\})\,.\end{aligned} \tag{2.52}$$

Multiplying $\bar u(P,S_z)\gamma^+\gamma_5$ $(\bar u(\bar P,S_z)\gamma^+\gamma_5)$ to the left (right) hand side of eqs. (2.49) and (2.51) and noting that SU(3) factors in eq. (2.46) and in eqs. (2.49), (2.51) are the same (i.e. $\epsilon^{ijl}\mathcal{P}_{inm}T^n_jP^m_l=\epsilon^{lij}(F_{L=1})_{min}T^n_jM^m_l,\epsilon^{ijl}(\mathcal{P}_Q)_{im}T^m_j(P_Q)_l=\epsilon^{lij}(F_{L=1})_{im}T^m_jM_l$) and hence can be factored out, we have

$$\begin{aligned}f(q^2)\;=\;&-\int\frac{dx_1d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2d^2k_{2\perp}}{2(2\pi)^3}\frac{1}{8P^+\widetilde{M}'_0\sqrt{2\;\beta^2_{23}(p_1\cdot P+m_1M_0)x'_2x_2x_3N_c}}\times\\&\times\mathrm{Tr}\left\{(\bar{\not P}+M_0)\gamma^+(\not p_2-\not p_3)(\not p'_2-m'_2)(\not p_1-m_1)\left[\not p_2-\not p_3-\frac{\bar P\cdot(p_2-p_3)}{M_0}\right]\right\}\times\\&\times\psi_{00}(x',k'_\perp)\;\Phi_{00}(\{x\},\{k_\perp\}),\end{aligned}$$

$$
\begin{aligned}
f_Q(q^2) = &-\int \frac{dx_1 d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{1}{8P^+\widetilde{M_0'}\sqrt{2\ \beta_{23}^2(p_1\cdot\bar{P}+m_1M_0)x_2'x_2x_3N_c}}\times\\
&\times \mathrm{Tr}\left\{(\bar{\not P}+M_0)\gamma^+(\not p_2-\not p_3)(\not p_2'-m_2')(\not p_1-m_1)\left[\not p_2-\not p_3-\frac{\bar{P}\cdot(p_2-p_3)}{M_0}\right]\right\}\times\\
&\times\psi_{00}(x',k_\perp')\ \Phi_{00}(\{x\},\{k_\perp\})\,. \qquad (2.53)
\end{aligned}
$$

Similarly by multiplying $\bar{u}(P,S_z)\gamma^+$ ($\bar{u}(\bar{P},S_z)\gamma^+$) to the left (right) hand side of eq. (2.52) and using the fact that $\epsilon^{ijl}(\mathcal{P}_Q)_{im}T_j^m(S_Q)_l = \epsilon^{lij}(F_{L=1})_{im}T_j^m M_l$, we arrive at

$$
\begin{aligned}
g_Q(q^2) = &-\int \frac{dx_1 d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{\widetilde{M_0'}}{16P^+M_0'\sqrt{3\ \beta_{23}^2\beta'^2(p_1\cdot\bar{P}+m_1M_0)x_2'x_2x_3N_c}}\times\\
&\times \mathrm{Tr}\left\{(\bar{\not P}+M_0)\gamma^+(\not p_2-\not p_3)(\not p_2'+m_2')(\not p_1-m_1)\left[\not p_2-\not p_3-\frac{\bar{P}\cdot(p_2-p_3)}{M_0}\right]\right\}\times\\
&\times\psi_{00}(x',k_\perp')\ \Phi_{00}(\{x\},\{k_\perp\})\,. \qquad (2.54)
\end{aligned}
$$

For a more explicit expression of above form factors we need to work out the corresponding traces. It is straightforward to obtain

$$
\begin{aligned}
&\frac{1}{4P^+}\mathrm{Tr}\left\{(\bar{\not P}+M_0)\gamma^+(\not p_2-\not p_3)(\not p_2'\mp m_2')(\not p_1-m_1)\left[\not p_2-\not p_3-\frac{\bar{P}\cdot(p_2-p_3)}{M_0}\right]\right\}=\\
&\quad= p_{23}^2[(p_1\cdot p_2'\pm m_1m_2')+x_2'(p_1\cdot\bar{P}+m_1M_0)-x_1(p_2'\cdot\bar{P}\mp m_2'M_0)]+\\
&\qquad+2x_{23}p_{23}\cdot p_1(p_2'\cdot\bar{P}\mp m_2'M_0)-2x_{23}p_{23}\cdot p_2'(p_1\cdot\bar{P}+m_1M_0)+\\
&\qquad+2p_{23}\cdot\bar{P}(x_1p_{23}\cdot p_2'-x_2'p_{23}\cdot p_1)-\frac{\bar{P}\cdot p_{23}}{M_0}\times\\
&\qquad\times\{p_{23}\cdot\bar{P}(\pm x_1m_2'+x_2'm_1)+x_{23}[M_0(p_1\cdot p_2'\pm m_1m_2')-m_1(p_2'\cdot\bar{P}\mp m_2'M_0)\mp\\
&\qquad\quad\mp m_2'(p_1\cdot\bar{P}+m_1M_0)]-p_{23}\cdot p_1(\pm m_2'+x_2'M_0)-p_{23}\cdot p_2'(m_1-x_1M_0)\}\,, \qquad (2.55)
\end{aligned}
$$

where use of $p_{23}\equiv p_2-p_3$, $x_{23}\equiv x_2-x_3$ has been made. To recast the above expression in terms of internal variables, it is useful to note that $p_1=p_1'$, $\bar{P}-\bar{P}'=p_2+p_3-p_2'=\bar{q}$, where $\bar{q}^+=q^+=0$, $\bar{q}_\perp=q_\perp$, and $\bar{q}^2=q^2=-q_\perp^2$, $\bar{q}\cdot(x_i^{(\prime)}\bar{P}^{(\prime)}-p_i^{(\prime)})=q_\perp\cdot k_{i\perp}^{(\prime)}$. We then obtain

$$
\begin{aligned}
&\bar{P}\cdot\bar{P}'=\frac{1}{2}(M_0^2+M_0'^2-q^2)\,,\qquad p_1\cdot p_2'\pm m_1m_2'=\frac{M_0'^2-(m_1\mp m_2')^2}{2}\,,\\
&p_1\cdot\bar{P}+m_1M_0=\frac{(m_1+x_1M_0)^2+k_{1\perp}^2}{2x_1}\,,\\
&p_2'\cdot\bar{P}\mp m_2'M_0=\frac{(m_2'\mp x_2'M_0)^2+(k_{1\perp}+q_\perp)^2}{2x_2'}\,,\\
&p_{23}^2=k_{23}^2=(k_2^+-k_3^+)(k_2^--k_3^-)-k_{23\perp}^2\,,\qquad \bar{P}\cdot p_{23}=M_0e_{23}=p_1\cdot p_{23}+m_2^2-m_3^2\,,\\
&\bar{q}\cdot p_{23}=-\bar{q}\cdot(x_{23}\bar{P}-p_{23})+x_{23}\bar{q}\cdot\bar{P}=-q_\perp\cdot k_{23\perp}+\frac{x_{23}}{2}(M_0^2-M_0'^2+q^2)\,,\\
&p_2'\cdot p_{23}=(p_2+p_3-\bar{q})\cdot p_{23}=\frac{2q_\perp\cdot k_{23\perp}-x_{23}(M_0^2-M_0'^2+q^2)+2(m_2^2-m_3^2)}{2}\,, \qquad (2.56)
\end{aligned}
$$

where uses of eq. (2.50), $e_{23} \equiv e_2 - e_3$ and $k_{23} \equiv k_2 - k_3$ have been made. Finally putting these together, we obtain

$$
\begin{aligned}
f(q^2) \;=\; & -\int \frac{dx_1 d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{1}{4x_2'\widetilde{M}_0'\sqrt{\beta_{23}^2[(m_1+x_1M_0)^2+k_{1\perp}^2]x_1x_2'x_2x_3N_c}} \times \\
& \times \Big\{x_1x_2'[M_0'^2-(m_1-m_2')^2](k_{23}^2-x_{23}M_0e_{23})+x_2'[(m_1+x_1M_0)^2+k_{1\perp}^2]\times \\
& \quad \times \{[k_{23}^2x_2'+x_{23}[-2q_\perp\cdot k_{23\perp}+x_{23}(M_0^2-M_0'^2+q^2)- \\
& \qquad -2(m_2^2-m_3^2)+m_2'e_{23}]\}+ \\
& \quad +x_1[(m_2'-x_2'M_0)^2+(k_{1\perp}+q_\perp)^2]\times \\
& \quad \times[-x_1k_{23}^2+2x_{23}(M_0e_{23}-m_2^2+m_3^2)+x_{23}m_1e_{23}]+ \\
& \quad +x_1x_2'e_{23}\{-2M_0e_{23}(x_1m_2'+x_2'm_1)+ \\
& \qquad\qquad +2(M_0e_{23}-m_2^2+m_3^2)(m_2'-x_2'M_0)+ \\
& \qquad\qquad +[2q_\perp\cdot k_{23\perp}-x_{23}(M_0^2-M_0'^2+q^2)+2(m_2^2-m_3^2)]\times \\
& \qquad\qquad \times(m_1+x_1M_0)\}\Big\}\times \\
& \times\psi_{00}(x',k_\perp')\;\Phi_{00}(\{x\},\{k_\perp\})\,, \\
f_Q(q^2) \;=\; & -\int \frac{dx_1 d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{1}{4x_2'\widetilde{M}_0'\sqrt{\beta_{23}^2[(m_1+x_1M_0)^2+k_{1\perp}^2]x_1x_2'x_2x_3N_c}} \times \\
& \times \Big\{x_1x_2'[M_0'^2-(m_1-m_2')^2](k_{23}^2-x_{23}M_0e_{23})+x_2'[(m_1+x_1M_0)^2+k_{1\perp}^2]\times \\
& \quad \times \{[k_{23}^2x_2'+x_{23}[-2q_\perp\cdot k_{23\perp}+x_{23}(M_0^2-M_0'^2+q^2) \\
& \qquad -2(m_2^2-m_3^2)+m_2'e_{23}]\}+ \\
& \quad +x_1[(m_2'-x_2'M_0)^2+(k_{1\perp}+q_\perp)^2]\times \\
& \quad \times[-x_1k_{23}^2+2x_{23}(M_0e_{23}-m_2^2+m_3^2)+x_{23}m_1e_{23}]++x_1x_2'e_{23}\times \\
& \quad \times\{-2M_0e_{23}(x_1m_2'+x_2'm_1)+2(M_0e_{23}-m_2^2+m_3^2)(m_2'-x_2'M_0)+ \\
& \qquad +[2q_\perp\cdot k_{23\perp}-x_{23}(M_0^2-M_0'^2+q^2)+2(m_2^2-m_3^2)](m_1+x_1M_0)\}\Big\}\times \\
& \times\psi_{00}(x',k_\perp')\;\Phi_{00}(\{x\},\{k_\perp\})\,, \\
g_Q(q^2) \;=\; & -\int \frac{dx_1 d^2k_{1\perp}}{2(2\pi)^3}\frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{\widetilde{M}_0'}{4x_2'M_0'\sqrt{6\beta_{23}^2\beta'^2[(m_1+x_1M_0)^2+k_{1\perp}^2]x_1x_2'x_2x_3N_c}} \times \\
& \times \Big\{x_1x_2'[M_0'^2-(m_1+m_2')^2](k_{23}^2-x_{23}M_0e_{23})+x_2'[(m_1+x_1M_0)^2+k_{1\perp}^2]\times \\
& \quad \times\;\{[k_{23}^2x_2'+x_{23}[-2q_\perp\cdot k_{23\perp}+x_{23}(M_0^2-M_0'^2+q^2)- \\
& \qquad 2(m_2^2-m_3^2)-m_2'e_{23}]\}+ \\
& \quad +x_1[(m_2'+x_2'M_0)^2+(k_{1\perp}+q_\perp)^2]\times \\
& \quad \times[-x_1k_{23}^2+2x_{23}(M_0e_{23}-m_2^2+m_3^2)+x_{23}m_1e_{23}]+x_1x_2'e_{23}\times \\
& \quad \times\;\{2M_0e_{23}(x_1m_2'-x_2'm_1)-2(M_0e_{23}-m_2^2+m_3^2)(m_2'+x_2'M_0)+ \\
& \qquad +[2q_\perp\cdot k_{23\perp}-x_{23}(M_0^2-M_0'^2+q^2)+2(m_2^2-m_3^2)](m_1+x_1M_0)\}\Big\}\times \\
& \times\psi_{00}(x',k_\perp')\;\Phi_{00}(\{x\},\{k_\perp\})\,. \qquad (2.57)
\end{aligned}
$$

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$m_{[ud]}$	$m_{[us]}$	m_u	m_s	m_c	β_{1c}	β_{1s}
0.40	0.56	0.23	0.45	1.3	0.58	0.48
$\beta_{23[qq']}$	β_π	β_K	β_D	β_{D_s}	$\beta_{D^*_{s0}}$	
0.38	0.35	0.377	0.456	0.478	0.340	

Table 1: The pentaquark masses and input parameters $m_{[qq']}$, m_q and β's (in units of GeV) appearing in the gaussian-type wave function (2.18).

$F^{\mathcal{P}\to M}$	$F(0)$ (GeV^2)	Λ_1 (GeV)	Λ_2 (GeV)
$f^{\Theta\to K}$	0.077	2.18	–
$f^{\Xi_{3/2}\to\pi}$	0.065	2.59	–
$f^{\Xi_{3/2}\to K}$	0.085	2.56	–
$f_c^{\Theta_c\to D}$	0.081	2.06	2.39
$f_c^{\Sigma_{5c}\to D_s}$	0.045	2.25	2.40
$g_c^{\Sigma_{5c}\to D^*_{s0}}$	0.024	2.64	2.32
$f_c^{\Xi_{5c}\to D_s}$	0.084	2.16	2.55
$g_c^{\Xi_{5c}\to D^*_{s0}}$	0.045	2.97	2.23

Table 2: The transition form factors for various pentaquark to meson transitions.

Numerical estimations of these form factors will be given in the next section.

3. Numerical results

3.1 Strong decays of pentaquark baryons

The input parameters $m_{[qq']}$, m_q, β_M, β_1 (for the anti-quark) and β_{23} (for the diquark pair) (see eq. (2.37)) that are relevant for our proposes are summarized in table 1. The quark masses and β_M's are taken from [60, 68] where the latter are obtained by fitting to the decay constants (cf. eq. (2.23)) as done in [68]. Note that our prediction $f_{D^*_{s0}} = 59\,\mathrm{MeV}$ [60] is consistent with the recent experimental result $f_{D^*_{s0}} \approx 47 - 73$ MeV [56, 69]. This supports a smaller value of $\beta_{D^*_{s0}}$ as shown in Table 1. Since the diquark pair acts like $\mathbf{3}_c$, the $\bar{q}$–$\{[ud][ud]\}$ system can be regarded as the analog of the heavy meson $\bar{q}$–q'. Therefore, it is plausible to assume that $\beta_{1c} : \beta_{1s} \sim \beta_D : \beta_K$ [39]. The $\beta_{23[qq']}$ parameter for the diquark pair is taken to be of order $\Lambda_{\rm QCD}$. The explicit numerical values of $\beta_{D,K}$ are taken from [60, 68]. As shown in [39], by using these input parameters, the obtained $\Sigma'_{5b} \to \Sigma'_{5c}$ transition form factors $f_1(0)$, $g_1(0)$ are close to their counterparts (in the sense of SU$_f(3)$ representation) in the $\Lambda_b \to \Lambda_c$ transition [70].

To proceed, we find that the momentum dependence of the form factors in the spacelike region can be well parameterized and reproduced in the three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - (q^2/\Lambda_1^2) + (q^2/\Lambda_2^2)^2} \tag{3.1}$$

for $\mathcal{P} \to M$ transitions. The parameters $\Lambda_{1,2}$, and $F(0)$ are first determined in the spacelike region. We then employ this parametrization to determine the physical form factors at $q^2 \geq 0$.

Table 2 gives various form factors obtained in the light-font approach. The from factors obtained from eqs. (2.53), (2.54) are fitted to the form of eq. (3.1). For pentaquarks, such as $\Sigma^0_{5c}(\bar{c}[ud][us])$, in which the two diquarks have different flavors, we need to average over $f(q^2)$ by applying $m_2 = m_{ud}$, $m_3 = m_{us}$ followed by an interchange of them in eqs. (2.53). We shall fit the form factors to the range of $-3\,\mathrm{GeV}^2 < q^2 \leq 0$ for light pentaquark transitions, and to the range of $-7\,\mathrm{GeV}^2 \leq q^2 \leq 0$ for heavy pentaquark transitions. Note that for $\Theta \to K$ and $\Xi_{3/2} \to \pi, K$ transition form factors a monopole form for their momentum dependence is adequate; an inclusion of the Λ_2 term will not affect the fit quality. For the case of heavy pentaquark transitions, since the form factors are fitted to a larger range of q^2, it is necessary to include the Λ_2 term in order to achieve a better fit.

Several remarks are in order: (i) It is interesting to note from table 2 that $\Theta \to K$ and $\Theta_c \to D$ form factors are very similar owing to the underlying spectator picture in which $\bar{s}$ and $\bar{c}$ are spectators. (ii) It is important to point out that the form factors of interest are indeed small as one can check explicitly that $f(q^2)/m^2_{\mathcal{P}}$, $f_Q(q^2)/m^2_{\mathcal{P}_Q}$, $g_Q(q^2)/m^2_{\mathcal{P}_Q} \ll 1$. The smallness of form factors is ascribed to the p-wave configuration of the two diquarks in an even-parity pentaquarks as it is necessary to bring the two diquarks close together to get involved interactions and produce an ordinary baryon with a s-wave quark configuration. As noted in passing, this phenomenon has been modelled in the present work by applying a local operator $\mathcal{O}_{\rm eff}$ for the $\phi\phi \to \mathcal{B}q$ transition. The mismatch in the orbital angular momentum configuration is the key physical reason for the smallness of these form factors. (iii) The $\mathcal{P}_c \to D^*_{s0}$ form factors are smaller than the $\mathcal{P}_c \to D_s$ ones by a factor of 2 owing to the smallness of $\beta_{D^*_{s0}}$. (iv) All the form factors are sensitive to β_{23}, for example, $f^{\Theta\to K}(0) = 0.077\,\mathrm{GeV}^2$ (see table 2) will be enhanced by 16 if β_{23} is changed from 0.38 GeV to 0.42 GeV. For pentaquark weak decays considered in [39], diquarks are spectators and hence weak decays are not sensitive to β_{23}. In this work, diquarks are no longer spectators and hence the strong transition form factors are sensitive to β_{23}. However, as the pentaquark decay rates are normalized to the $\Theta \to NK$ one, the β_{23} dependence will be reduced.

With the numerical results of strong transition form factors given in table 2, we are ready to estimate the corresponding strong decays. The $\mathcal{P} \to \mathcal{B}M$ decay amplitudes are given by

$$\mathcal{A}(\mathcal{P} \to \mathcal{B}M) = \bar{u}(P_{\mathcal{B}}, S'_z)(A + iB\gamma_5)u(P_{\mathcal{P}}, S_z)\,, \tag{3.2}$$

with ($\kappa \equiv g_{2\rm eff}/M^2$)

$$\begin{aligned}
B[\mathcal{P}(\overline{\mathbf{10}}) \to \mathcal{B}P(\mathbf{8})] &= \epsilon^{ijk}\mathcal{P}_{imn}T^m_j P^n_k\ \kappa f(m^2_{\mathcal{B}})\,,\\
B[\mathcal{P}_Q(\bar{\mathbf{6}}) \to \mathcal{B}P_Q(\mathbf{3})] &= \epsilon^{ijk}(\mathcal{P}_Q)_{im}T^m_j(P_Q)_k\ \kappa f_Q(m^2_{\mathcal{B}})\,,\\
A[\mathcal{P}_Q(\bar{\mathbf{6}}) \to \mathcal{B}S_Q(\mathbf{3})] &= -\epsilon^{ijk}(\mathcal{P}_Q)_{im}T^m_j(P_Q)_k\ \kappa g_Q(m^2_{\mathcal{B}})\,,\\
A[\mathcal{P}(\overline{\mathbf{10}}) \to \mathcal{B}P(\mathbf{8})] &= A[\mathcal{P}_Q(\bar{\mathbf{6}}) \to \mathcal{B}P_Q(\mathbf{3})] = B[\mathcal{P}_Q(\bar{\mathbf{6}}) \to \mathcal{B}P_Q(\mathbf{3})] = 0\,,
\end{aligned} \tag{3.3}$$

followed from eqs. (2.43) and (2.46). The explicit expression of the Clebsch-Gordan coefficients in eq. (3.3) can be found in the appendix. For the decay modes under consideration,

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the non-vanishing amplitudes read

$$\begin{aligned}
B(\Theta \to pK^0) &= -B(\Theta \to nK^+) = \kappa f^{\Theta\to K}(m_N^2)\,,\\
B(\Xi_{3/2}^{--} \to \Xi^-\pi^-) &= \kappa f^{\Xi_{3/2}\to\pi}(m_\Xi^2)\,,\\
B(\Xi_{3/2}^{--} \to \Sigma^- K^-) &= \kappa f^{\Xi_{3/2}\to K}(m_\Sigma^2)\,,\\
B(\Theta_c^0 \to pD^-) &= \kappa f_c^{\Theta_c\to D}(m_p^2)\,,\\
B(\Sigma_{5c}^0 \to pD_s^-) &= -\frac{\kappa}{\sqrt{2}}\, f_c^{\Sigma_{5c}\to D_s}(m_p^2)\,,\\
A(\Sigma_{5c}^0 \to pD_{s0}^{*-}) &= \frac{\kappa}{\sqrt{2}}\, g_c^{\Sigma_{5c}\to D_{s0}^*}(m_p^2)\,,\\
B(\Xi_{5c}^0 \to \Sigma^+ D_s^-) &= -\kappa f_c^{\Xi_{5c}\to D_s}(m_\Sigma^2)\,,\\
A(\Xi_{5c}^0 \to \Sigma^+ D_{s0}^{*-}) &= \kappa g_c^{\Xi_{5c}\to D_{s0}^*}(m_\Sigma^2)\,,
\end{aligned} \tag{3.4}$$

where the quark flavor content of the sextet charmed pentaquarks is explained in [39]. The decay rate can be evaluated via [70]

$$\Gamma(\mathcal{P} \to \mathcal{B}M) = \frac{p_c}{8\pi}\left[\frac{(m_\mathcal{P}+m_\mathcal{B})^2 - m_M^2}{m_\mathcal{P}^2}|A|^2 + \frac{(m_\mathcal{P}-m_\mathcal{B})^2 - m_M^2}{m_\mathcal{P}^2}|B|^2\right], \tag{3.5}$$

where p_c is the c.m. momentum of the final state in the pentaquark rest frame.

By fitting to $\Gamma(\Theta \to pK^0) = \frac{1}{2}\Gamma(\Theta) \simeq 0.5\,\mathrm{MeV}$, we obtain $\kappa f(m_N^2) \simeq 0.97$. Using the result of $f(m_N^2) = 0.095\,\mathrm{GeV}^2$ from table 2, it follows that $\kappa \equiv g_{2\mathrm{eff}}/M^2 \simeq 10.2\,\mathrm{GeV}^{-2}$, where M is a characteristic scale of the $\phi\phi \to \mathcal{B}q$ transition. Taking $M \simeq 1\,\mathrm{GeV}$, we have $g_{2\mathrm{eff}} \simeq 10.2$, which is slightly smaller than the strong πNN coupling $g_{\pi NN} \sim 14$. This suppression could be understood as the cost to pay for breaking one of the diquarks into two quarks.

With the effective strong coupling in a reasonable size, it is now plausible to ascribe the narrow width of Θ^+ to the suppressed transition form factors ($f^{\Theta\to K}/m_\Theta^2 \ll 1$). As noted in passing, this suppression arises from bringing the two diquarks in a p-wave configuration close together to form a final state baryon in the s-wave quark configuration. This observation is close to the finding of [52], where the narrowness of the Θ decay width is due to a large tunneling suppression of a quark between a pair of diquarks separated by the p-wave centrifugal potential. Furthermore, based on the Jaffe-Wilczek picture for pentaquarks, it is argued in [54] that a strong dynamical suppression of the pentaquark width requires a rather asymmetric "peanut" structure in which the strange antiquark is in the center while the two extended diquarks rotate around [54]. The decay width is decreasing with the sizes of the diquark. These features are not inconsistent with our picture of treating the diquark as an effective degree of freedom. The p-wave configuration of the diquark pair is very important in resulting a small decay width.[4] In particular, the choice of $\beta_{1s} > \beta_{23[qq']}$ in table 1 corresponds to the "peanut" structure advocated in [54]. Furthermore, although we take the diquark as an effective degree of freedom, it is not necessary to be a point-like

[4]However, it was proposed recently in [55] that a pentaquark with an internal s-wave structure (an s-wave between two diquarks with different chirality) becomes stable in the chiral limit and hence may explain the tiny decay width.

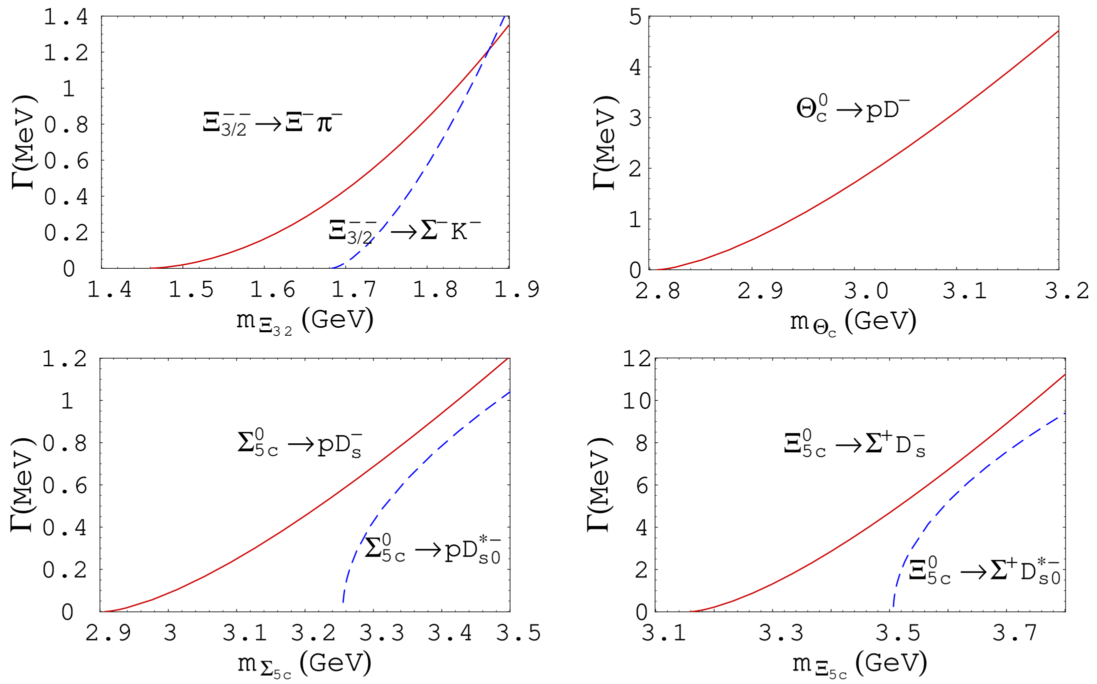

Figure 2: Decay rates (in units of MeV) for $\Xi^{--}_{3/2} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Theta^0_c \to pD^-$, $\Sigma_{5c} \to pD^-_s, pD^{*-}_{s0}$ and $\Xi^0_{5c} \to \Sigma^+D^-_s, \Sigma^+D^{*-}_{s0}$ as a function of the pentaquark mass. These rates are normalized to $\Gamma(\Theta^+) = 2\Gamma(\Theta^+ \to pK^0) = 1\,\text{MeV}$.

object. In fact, as the size of the diquark increases, it becomes easier to break it into two quarks before fragmenting into the final-state baryon and meson. The narrow pentaquark decay width favors a small-size diquark, which is consistent with our treatment of taking it effectively as a fundamental object.

Treating κ to be approximately universal, we can estimate the strong decay rates of $\Xi^{--}_{3/2} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Theta^0_c \to pD^-$, $\Sigma^0_{5c} \to pD^-_s, pD^{*}_{s0}$ and $\Xi^0_{5c} \to \Sigma^+D^-_s, \Sigma^+D^{*-}_{s0}$. In figure 2, we show these rates normalized to $\Gamma(\Theta) = 2\Gamma(\Theta \to pK^0) = 1$ MeV as a function of the pentaquark mass.[5] It is clear that $\Xi^{--}_{3/2} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Sigma^0_{5c} \to pD^-_s, pD^{*-}_{s0}$ and $\Theta^0_c \to pD^-$ decay rates are of order a few MeV, while $\Xi^0_{5c} \to \Sigma^+D^-_s, \Sigma^+D^{*-}_{s0}$ decay rates are of order tens of MeV. In particular, by taking $m_{\Theta_c} \simeq 3.1\,\text{GeV}$ as observed by H1 collaboration [36], we obtain $\Gamma(\Theta^0_c \to pD^-) \simeq 3.1\,\text{MeV}$, which is consistent with the observed width of $\Gamma(\Theta_c) = 12 \pm 3\,\text{MeV}$ [36]. Taking $m_{\Xi^{--}_{3/2}} = 1862 \pm 2\,\text{MeV}$ as measured by NA49 [14], we obtain $\Gamma(\Xi^{--}_{3/2} \to \Sigma^-K^-) \simeq 1.07\,\text{MeV}$ and $\Gamma(\Xi^{--}_{3/2} \to \Xi^-\pi^-) \simeq 1.13$ MeV, which are again consistent with the observed width of $\Gamma(\Xi^{--}_{3/2}) \leq 18\,\text{MeV}$ [14]. Our estimation for the ratio $\Gamma(\Xi^{--}_{3/2} \to \Xi^-\pi^-)/\Gamma(\Theta^+ \to pK^0) \simeq 2.2$ is several times smaller than that of [50] but close to the estimate made in [49]. Note that the ratio $\Gamma(\Xi^{--}_{3/2} \to \Sigma^-K^-)/\Gamma(\Xi^{--}_{3/2} \to \Xi^-\pi^-) \simeq 0.94$ is 50% larger than that obtained in [49] based solely on the phase space consideration. The enhancement is due to the form factor ratio $f^{\Xi_{3/2}\to K}(m^2_\Sigma)/f^{\Xi_{3/2}\to\pi}(m^2_\Xi) = 1.23$ (cf. table 2) obtained in the LF calculation.

[5]In this estimation the dependence of the pentaquark mass in rates are explicitly shown in eq. (3.5) with A and B terms being kept fixed in the mass range under consideration.

So far we have focused only on the strong decays of the pentaquarks into an octet baryon and a pseudoscalar meson. For the decay into a vector meson, it involves an additional unknown tensor coupling which is calculable within our light-front framework. Moreover, in the heavy quark limit $\mathcal{P}_Q \to M_Q\mathcal{B}$ and $\mathcal{P}_Q \to M_Q^*\mathcal{B}$ are governed by the same strong coupling constant. Indeed, heavy quark symmetry leads to the relation $\Gamma(\mathcal{P}_Q \to M_Q^*\mathcal{B}) = 3\Gamma(\mathcal{P}_Q \to M_Q\mathcal{B})$ [71]. Since $\Theta_c \to D^{*-}p$ has been observed by H1 [36], it will be interesting to measure the rate of $\Theta_c \to D^- p$ to test heavy quark symmetry. Our result $\Gamma(\Theta_c^0 \to pD^-) \simeq 3.1$ MeV will imply $\Gamma(\Theta_c^0 \to pD^{*-}) \sim 9$ MeV. With $\Gamma(\Theta_c^0 \to \overline{D}^{(*)0}n) \simeq \Gamma(\Theta_c^0 \to D^{(*)-}p)$ we expect $\Gamma(\Theta_c) \simeq 20\,\mathrm{GeV}$. This is in accordance with the observed width of $\Gamma(\Theta_c^0) = 12 \pm 3\,\mathrm{MeV}$ [36].

It is interesting to note that although the $\mathcal{P}_c \to D_{s0}^*$ form factor is smaller than the $\mathcal{P}_c \to D_s$ one by a factor of 2, the decay rate for the D_{s0}^* production is comparable to that for D_s. This can be understood from the parity consideration. Since $\mathcal{P}_c$, $\mathcal{B}$, D_{s0}^* are parity even, the final state $\mathcal{B}D_{s0}^*$ in $\mathcal{P}_c$ decay can have a s-wave configuration, while the $\mathcal{B}D_s$ state must be in a p-wave or higher odd-wave configuration, whose rate is suppressed near the threshold. However, such a suppression is absent in the final state composed of an even-parity meson and an even-parity baryon and hence the decay $\mathcal{P}_c \to \mathcal{B}D_{s0}^*$ can have a sizable decay rate even its transition form factor is suppressed. This is the reason why we have $\Gamma(\Sigma_{5c}^0 \to pD_{s0}^{*-}) \simeq \Gamma(\Sigma_{5c}^0 \to pD_s^-)$ and $\Gamma(\Xi_{5c}^0 \to \Sigma^+ D_{s0}^{*-}) \simeq \Gamma(\Xi_{5c}^0 \to \Sigma^+ D_s^-)$ (see figure 2), provided that these strong decays are kinematically allowed. As this is closely related to the even-parity nature of these pentaquarks, the ratio of $\Gamma(\mathcal{P}_c \to \mathcal{B}D_{s0}^*)/\Gamma(\mathcal{P}_c \to \mathcal{B}D_s)$ provides for a useful way for verifying the parity of the charmed pentaquark. For example, a completely opposite pattern — $\Gamma(\Sigma_{5c} \to pD_{s0}^{*-}) \ll \Gamma(\Sigma_{5c} \to pD_s)$ and $\Gamma(\Xi_{5c} \to \Sigma D_{s0}^{*-}) \ll \Gamma(\Xi_{5c} \to \Sigma D_s)$ – is expected for odd-parity pentaquarks Σ_{5c} and Ξ_{5c}. It should be remarked that $m_{D_{s0}^*} \simeq 2.317\,\mathrm{GeV}$ [57, 56] is substantially smaller than expected from the quark model and hence the $D_{s0}^*\mathcal{B}$ threshold is close to the $D_s\mathcal{B}$ one, rendering the production of the former easier than naive anticipation.

Finally, it is worth commenting that $\mathcal{P}_c$ can be produced in B decays via $B \to \mathcal{P}_c\bar{\mathcal{B}}$ such as $B^+ \to \Theta_c^0\bar{\Delta}^+$ and $B^0 \to \Theta_c^0\bar{p}\pi^+$ [72, 73]. Theoretically, it is difficult to estimate their branching ratios. Nevertheless, the measured branching ratios by Belle for charmful baryonic B decays [74], $\mathcal{B}(\overline{B}^0 \to \Lambda_c^+\bar{p}) = (2.2^{+0.6}_{-0.5} \pm 0.3 \pm 0.6) \times 10^{-5}$ and $\mathcal{B}(B^- \to \Lambda_c^+\bar{p}\pi^-) = (1.87^{+0.43}_{-0.40} \pm 0.28 \pm 0.49) \times 10^{-4}$, provide some useful cue. Since a production of the pentaquark needs one more pair of $q\bar{q}$ compared to the normal baryon, it is plausible to expect that the branching ratios of $B^+ \to \Theta_c^0\bar{\Delta}^+$ and $B^0 \to \Theta_c^0\bar{p}\pi^+$ are at most of order 10^{-6} and 10^{-5}, respectively. Hence, they may be barely reachable at B factories. Nevertheless, one can search for $\mathcal{P}_c$ through $B \to \mathcal{P}_c\bar{\mathcal{B}}' \to (D\mathcal{B})\bar{\mathcal{B}}'$, $(D_s\mathcal{B})\bar{\mathcal{B}}'$, $(D_{s0}^*\mathcal{B})\bar{\mathcal{B}}'$ decays.

4. Conclusions

Assuming the two diquark structure for the pentaquark as advocated in the Jaffe-Wilczek model, we study the strong decays of pentaquark baryons using the light-front approach in conjunction with the spectator approximation. The main conclusions are as follows.

1. In the Jaffe-Wilczek model, the diquark pairs in the light antidecuplet and heavy antisextet pentaquark baryons are in a p-wave configuration. To describe their strong decays, the two diquarks must interact to produce an ordinary baryon with a s-wave quark configuration. This phenomenon has been modelled in the present work by applying a local operator $\mathcal{O}_{\rm eff}$ for the $\phi\phi \to \mathcal{B}q$ transition. With a reasonable (and unsuppressed) strong coupling of $\mathcal{O}_{\rm eff}$ we see that the mismatch in the orbital angular momentum configuration is the key physical reason for the narrowness of the pentaquark decay width.

2. Treating the subprocess $\phi\phi \to \mathcal{B}q$ to be approximately universal as suggested by the spectator picture, we estimate the strong decays $\Xi_{3/2}^{--} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Theta_c^0 \to pD^-$, $\Sigma_{5c} \to pD_s^-, pD_{s0}^{*-}$ and $\Xi_{5c}^0 \to \Sigma^+D_s^-, pD_{s0}^{*-}$ by normalizing to the Θ^+ width. We find that $\Xi_{3/2}^{--} \to \Xi^-\pi^-, \Sigma^-K^-$, $\Sigma_{5c} \to pD_s^-, pD_{s0}^{*-}$ and $\Theta_c^0 \to pD^-$decay rates are of the order of a few MeV, while and $\Xi_{5c}^0 \to \Sigma^+D_s^-, \Sigma^+D_{s0}^{*-}$ decay rates are of order tens of MeV. If we take $m_{\Xi_{3/2}^{--}} = 1862 \pm 2\,\mathrm{MeV}$ as observed by NA49 [14], we have $\Gamma(\Xi_{3/2}^{--} \to \Xi^-\pi^-)/\Gamma(\Theta^+ \to pK^0) \simeq 2.2$ which is consistent with the observed width of $\Gamma(\Xi_{3/2}^{--}) \leq 18\,\mathrm{MeV}$ [14].

3. Since the mass of the scalar meson D_{s0}^{*-} is observed to be lighter than expected, we also study $\mathcal{P}_c \to D_{s0}^*\mathcal{B}$ decays in addition to $\mathcal{P}_c \to D_s\mathcal{B}$ decays. The former modes are enhanced (or unsuppressed) due to the even-parity nature of $\mathcal{P}_c, \mathcal{B}$ and D_{s0}^*. In particular, the experimental study of the ratio of $\Gamma(\mathcal{P}_c \to \mathcal{B}D_{s0}^*)/\Gamma(\mathcal{P}_c \to \mathcal{B}D_s)$ could be very useful for verifying the parity of the sextet charmed pentaquark $\mathcal{P}_c$. It is expected to be of order unity for an even parity $\mathcal{P}_c$ and much less than one for an odd parity one.

4. We also pointed out the possibility to search for $\mathcal{P}_c$ through $B \to \mathcal{P}_c\bar{\mathcal{B}}' \to (D\mathcal{B})\bar{\mathcal{B}}'$, $(D_s\mathcal{B})\bar{\mathcal{B}}'$, $(D_{s0}^*\mathcal{B})\bar{\mathcal{B}}'$ decays.

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A. Clebsch-Gordan coefficients for pentauqrks, octet baryons and mesons

In this appendix we specify the pentaquark, octet baryon and meson $\mathrm{SU}_f(3)$ quantum numbers. For anti-decuplet pentaquarks, we use the totally symmetric tensor P_{ijk} satisfying the normalization condition $P_{ijk}P^{ijk} = 1$

$$P_{333} = \Theta^+\,,$$
$$P_{133} = \frac{1}{\sqrt{3}}N_{\overline{10}}^0\,, \qquad P_{233} = \frac{1}{\sqrt{3}}N_{\overline{10}}^+\,,$$

$$
\begin{aligned}
P_{113} &= \frac{1}{\sqrt{3}}\Sigma^{-}_{\overline{10}}\,, \qquad P_{123} = \frac{1}{\sqrt{6}}\Sigma^{0}_{\overline{10}}\,, \qquad P_{223} = \frac{1}{\sqrt{3}}\Sigma^{+}_{\overline{10}}\,, \\
P_{111} &= \Xi^{--}_{3/2}\,, \qquad P_{112} = \Xi^{-}_{3/2}\,, \qquad P_{122} = \Xi^{0}_{3/2}\,, \qquad P_{222} = \Xi^{+}_{3/2}\,.
\end{aligned} \tag{A.1}
$$

Note that $(F_{L=1})_{ijk} = P_{ijk}$. Anti-sextet heavy pentaquarks are described by the totally symmetric tensor $(\mathcal{P}_Q)_{ij}$. In the case of charm pentaquarks, we have:

$$
\begin{aligned}
(\mathcal{P}_c)_{33} &= \Theta^0_c\,, \\
(\mathcal{P}_c)_{13} &= \frac{1}{\sqrt{2}}\Sigma^{-}_{5c}\,, \qquad (\mathcal{P}_c)_{23} = \frac{1}{\sqrt{2}}\Sigma^{0}_{5c}\,, \\
(\mathcal{P}_c)_{11} &= \Xi^{--}_{5c}\,, \qquad (\mathcal{P}_c)_{12} = \frac{1}{\sqrt{2}}\Xi^{-}_{5c}\,, \qquad (\mathcal{P}_c)_{22} = \Xi^{0}_{5c}\,.
\end{aligned} \tag{A.2}
$$

The $\mathrm{SU}_f(3)$ structure of octet baryons and mesons are represented by $\mathcal{B} = T$ and M:

$$
\mathcal{B} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix} M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}. \tag{A.3}
$$

For example, for a final state p and K^0 in the Θ^+ decay, we need to use $T^3_1 = p$, $M^3_2 = K^0$. Heavy mesons $(\overline{D}^0, D^-, D^-_s)$ transform like a triplet (u, d, s) under $\mathrm{SU}_f(3)$.

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